

Formulas

Conversion of the 13-week Treasury Bill auction clearing price to an ACT/360 simple interest rate:

$$r_t = \frac{360}{L} \times \left(\frac{100}{P_t} - 1 \right) \quad (1)$$

Where,

r_t = ACT/360 simple interest rate at issue date t

P_t = 13-week Treasury Bill auction clearing price at issue date t

L = number of days from (and including) the issue date of the 13-week Treasury bill to (and excluding) the maturity date of the 13-week Treasury bill

For example, on September 10, 2012, Treasury auctioned a 91-day Treasury bill at a price of 99.974722. The converted ACT/360 simple interest rate is:

$$\frac{360}{91} \times \left(\frac{100}{99.974722} - 1 \right) = 0.0010003 = 0.10003\% .$$

This rate is likely to be truncated or rounded to the nearest tenth of a basis point but the final decisions on precision will be determined later.

Assuming a \$100 notional, the formula below illustrates how to arrive at the *Dirty Price* at settlement date T_0 , the amount of money due at settlement.

$$\begin{aligned} \text{Dirty Price} &= \frac{\frac{1}{360} \sum_{T_1 \leq t < T_i} \max(r_t + s, 0)}{1 + \frac{1}{360} \sum_{T_0 \leq t < T_1} (r_t + m)} \\ &\quad + \frac{\frac{1}{360} \sum_{T_1 \leq t < T_2} \max(r_t + s, 0)}{[1 + \frac{1}{360} \sum_{T_0 \leq t < T_1} (r_t + m)] \times [1 + \frac{1}{360} \sum_{T_1 \leq t < T_2} (r_t + m)]} \\ &\quad + \dots \\ &\quad + \frac{1 + \frac{1}{360} \sum_{T_{N-1} \leq t < T_N} \max(r_t + s, 0)}{[1 + \frac{1}{360} \sum_{T_0 \leq t < T_1} (r_t + m)] \times \dots \times [1 + \frac{1}{360} \sum_{T_{N-1} \leq t < T_N} (r_t + m)]} \end{aligned} \quad (2)$$

Where,

T_0 = settlement date

T_{-1} = start of the *Interest Accrual Period*

r_t = *Index Rate* on day t ¹

s = *Spread*

m = *Discount Margin*

Note,

$$T_{-1} \leq T_0$$

When $T_{-1} < T_0$, there will be *Accrued Interest*

The next coupon payment is T_1

All other coupon payments (T_2, T_3, \dots, T_N) continue until maturity with a quarterly *Frequency of Interest Payments*

Day Count convention is ACT/360

Reset Frequency is daily

$\max(r_t + s, 0)$ because of the *Minimum Interest Rate*

Define the *Accrued Interest* as the accrual amount as of the settlement date T_0 , that is,

$$\text{Accrued Interest} = 100 \times \frac{1}{360} \sum_{T_{-1} \leq t < T_0} \max(r_t + s, 0) \quad (3)$$

The *Clean Price* is derived by subtracting the *Accrued Interest* from the *Dirty Price*.²
That is,

$$\text{Clean Price} = \text{Dirty Price} - \text{Accrued Interest} \quad (4)$$

An example calculation can be found on the Bureau of the Public Debt's website at
<http://www.treasurydirect.gov/instit/statreg/auctreg/DMCalc.xlsm>.

¹ *Index Rate* r_t beyond the settlement date T_0 is fixed at the value obtained from the last available good fixing.

² This methodology does not enable a *Clean Price* of par any time when the *Discount Margin* equals the *Spread* because (1) the *Accrued Interest* is not discounted to the settlement date, and/or (2) when the *Minimum Interest Rate* is binding.